

Physics 4A

Chapter 4: Kinematics in Two Dimensions

GENERAL PRINCIPLES

The instantaneous velocity

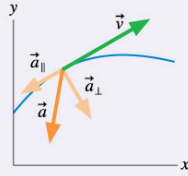
$$\vec{v} = d\vec{r}/dt$$

is a vector tangent to the trajectory.

The instantaneous acceleration is

$$\vec{a} = d\vec{v}/dt$$

\vec{a}_{\parallel} , the component of \vec{a} parallel to \vec{v} , is responsible for change of speed. \vec{a}_{\perp} , the component of \vec{a} perpendicular to \vec{v} , is responsible for change of direction.



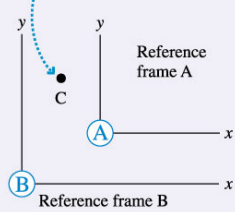
Relative Motion

If object C moves relative to reference frame A with velocity \vec{v}_{CA} , then it moves relative to a different reference frame B with velocity

$$\vec{v}_{CB} = \vec{v}_{CA} + \vec{v}_{AB}$$

where \vec{v}_{AB} is the velocity of A relative to B. This is the Galilean transformation of velocity.

Object C moves relative to both A and B.



© 2017 Pearson Education, Inc.

IMPORTANT CONCEPTS

Uniform Circular Motion

Angular velocity $\omega = d\theta/dt$.

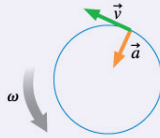
v_t and ω are constant:

$$v_t = \omega r$$

The centripetal acceleration points toward the center of the circle:

$$a = \frac{v^2}{r} = \omega^2 r$$

It changes the particle's direction but not its speed.



Nonuniform Circular Motion

Angular acceleration $\alpha = d\omega/dt$.

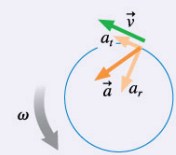
The radial acceleration

$$a_r = \frac{v^2}{r} = \omega^2 r$$

changes the particle's direction. The tangential component

$$a_t = \alpha r$$

changes the particle's speed.



© 2017 Pearson Education, Inc.

APPLICATIONS

Kinematics in two dimensions

If \vec{a} is constant, then the x- and y-components of motion are independent of each other.

$$x_f = x_i + v_{ix} \Delta t + \frac{1}{2} a_x (\Delta t)^2$$

$$y_f = y_i + v_{iy} \Delta t + \frac{1}{2} a_y (\Delta t)^2$$

$$v_{fx} = v_{ix} + a_x \Delta t$$

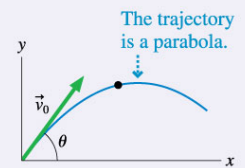
$$v_{fy} = v_{iy} + a_y \Delta t$$

Projectile motion is motion under the influence of only gravity.

MODEL Model as a particle launched with speed v_0 at angle θ .

VISUALIZE Use coordinates with the x-axis horizontal and the y-axis vertical.

SOLVE The horizontal motion is uniform with $v_x = v_0 \cos \theta$. The vertical motion is free fall with $a_y = -g$. The x and y kinematic equations have the same value for Δt .



Circular motion kinematics

$$\text{Period } T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$$

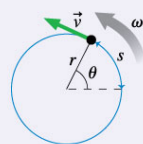
$$\text{Angular position } \theta = \frac{s}{r}$$

Constant angular acceleration

$$\omega_f = \omega_i + \alpha \Delta t$$

$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2$$

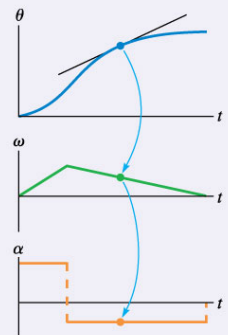
$$\omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta$$



Circular motion graphs and kinematics are analogous to linear motion with constant acceleration.

Angle, angular velocity, and angular acceleration are related graphically.

- The angular velocity is the slope of the angular position graph.
- The angular acceleration is the slope of the angular velocity graph.



© 2017 Pearson Education, Inc.

Conceptual Questions and Example Problems from Chapter 4

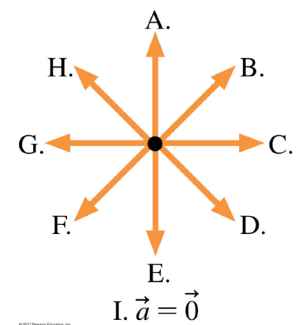
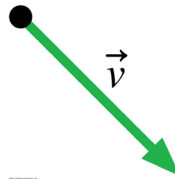
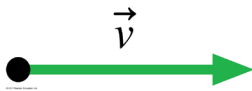
Conceptual Question 4.5

For a projectile, which of the following quantities are constant during flight: x , y , r , v_x , v_y , v , a_x , a_y ? Which of these quantities are zero throughout the flight?

4.5. For a projectile, only v_x , a_x , and a_y are constant during the flight. Since the acceleration $a_y = -g$ is down, $a_x = 0$ and v_x is constant. The nonzero a_y is constantly changing v_y , so the total speed $v = \sqrt{v_x^2 + v_y^2}$ changes as well. The positions x and y change, so $r = \sqrt{x^2 + y^2}$ changes, too. Only a_x is zero throughout the flight.

Problem 4.4

At this instant, the particle on the left is speeding up and curving upward. What is the direction of its acceleration?



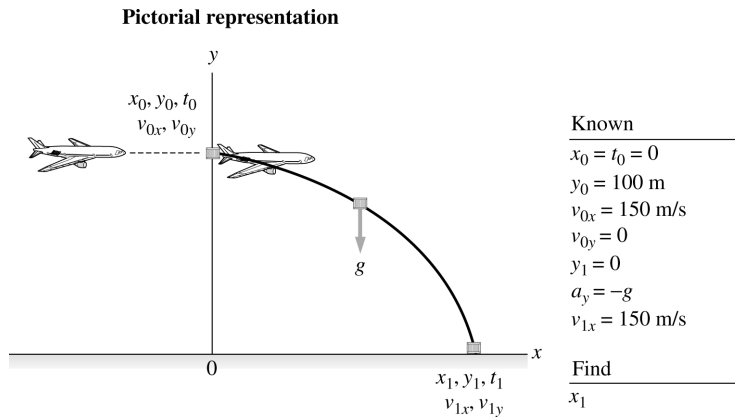
4.4. Solve: To make the particle speed up the acceleration needs to have a component that is in the direction of the velocity. To make the particle curve upward the acceleration must have a component upward. So the answer is B.

Problem 4.13

A supply plane needs to drop a package of food to scientists working on a glacier in Greenland. The plane flies 100 m above the glacier at a speed of 150 m/s. How far short of the target should it drop the package?

4.13. Model: We will use the particle model for the food package and the constant-acceleration kinematic equations of motion.

Visualize:



Solve: For the horizontal motion,

$$x_1 = x_0 + v_{0x}(t_1 - t_0) + \frac{1}{2}a_x(t_1 - t_0)^2 = 0 \text{ m} + (150 \text{ m/s})(t_1 - 0 \text{ s}) + 0 \text{ m} = (150 \text{ m/s})t_1$$

We will determine t_1 from the vertical y -motion as follows:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$$

$$\Rightarrow 0 \text{ m} = 100 \text{ m} + 0 \text{ m} + \frac{1}{2}(-9.8 \text{ m/s}^2)t_1^2 \Rightarrow t_1 = \sqrt{\frac{200 \text{ m}}{9.8 \text{ m/s}^2}} = 4.518 \text{ s} \approx 4.5 \text{ s}$$

From the above x -equation, the displacement is $x_1 = (150 \text{ m/s})(4.518 \text{ s}) = 678 \text{ m} \approx 680 \text{ m}$.

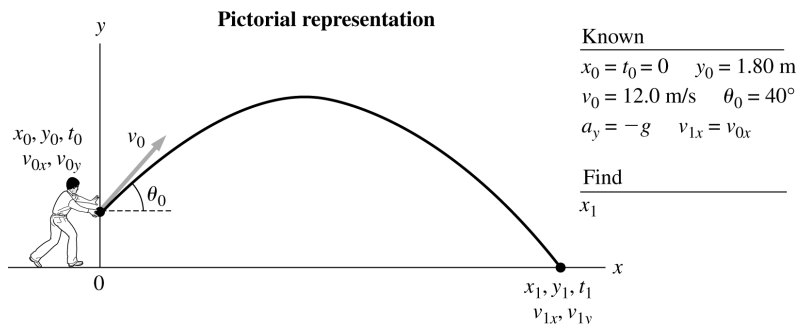
Assess: The horizontal distance of 678 m covered by a freely falling object from a height of 100 m and with an initial horizontal velocity of 150 m/s ($\approx 335 \text{ mph}$) is reasonable.

Problem 4.15

In the Olympic shotput event, an athlete throws the shot with an initial speed of 12.0 m/s at a 40.0° angle from the horizontal. The shot leaves her hand at a height of 1.80 m above the ground. How far does the shot travel?

4.15. Model: Assume the particle model and motion under constant-acceleration kinematic equations in a plane.

Visualize:



Solve: (a) Using $y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2$,

$$0 \text{ m} = 1.80 \text{ m} + v_0 \sin 40^\circ(t_1 - 0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(t_1 - 0 \text{ s})^2$$

$$= 1.80 \text{ m} + (7.713 \text{ m/s})t_1 - (4.9 \text{ m/s}^2)t_1^2 \Rightarrow t_1 = -0.206 \text{ s} \text{ and } 1.780 \text{ s}$$

The negative value of t_1 is unphysical for the current situation. Using $t_1 = 1.780$ s and $x_1 = x_0 + v_{0x}(t_1 - t_0)$, we get

$$x_1 = 0 + (v_0 \cos 40^\circ \text{ m/s})(1.780 \text{ s} - 0 \text{ s}) = (12.0 \text{ m/s}) \cos 40^\circ (1.78 \text{ s}) = 16.36 \text{ m} \approx 16.4 \text{ m}$$

(b) We can repeat the calculation for each angle. A general result for the flight time at angle θ is

$$t_1 = \left(12 \sin \theta + \sqrt{144 \sin^2 \theta + 35.28} \right) / 9.8 \text{ s}$$

and the distance traveled is $x_1 = (12.0) \cos \theta \times t_1$. We can put the results in a table.

θ	t_1	x_1
40.0°	1.780 s	16.36 m
42.5°	1.853 s	16.39 m
45.0°	1.923 s	16.31 m
47.5°	1.990 s	16.13 m

Maximum distance is achieved at $\theta \approx 42.5^\circ$.

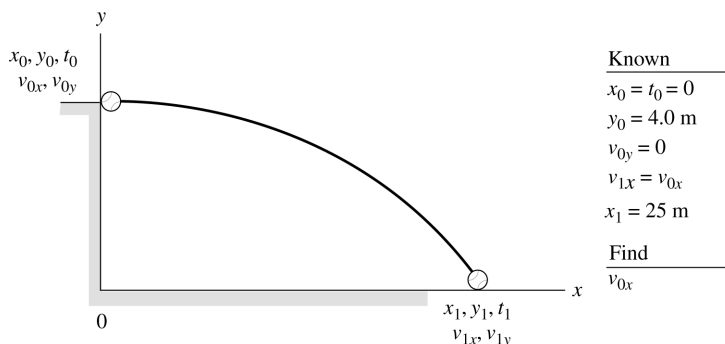
Assess: The well-known “fact” that maximum distance is achieved at 45° is true only when the projectile is launched and lands at the *same* height. That isn’t true here. The extra 0.03 m = 3 cm obtained by increasing the angle from 40.0° to 42.5° could easily mean the difference between first and second place in a world-class meet.

Problem 4.17

A baseball player friend of yours wants to determine his pitching speed. You have him stand on a ledge and throw the ball horizontally from a distance of 4.0 m above the ground. The ball lands 25 m away. What is his pitching speed?

4.17. Model: The particle model for the ball and the constant-acceleration equations of motion in a plane are assumed.

Visualize:



Solve: **(a)** The time for the ball to fall is calculated as follows:

$$y_1 = y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2} a_y (t_1 - t_0)^2$$

$$\Rightarrow 0 \text{ m} = 4 \text{ m} + 0 \text{ m} + \frac{1}{2} (-9.8 \text{ m/s}^2) (t_1 - 0 \text{ s})^2 \Rightarrow t_1 = 0.9035 \text{ s}$$

Using this result for the horizontal velocity:

$$x_1 = x_0 + v_{0x}(t_1 - t_0) \Rightarrow 25 \text{ m} = 0 \text{ m} + v_{0x}(0.9035 \text{ s} - 0 \text{ s}) \Rightarrow v_{0x} = 27.7 \text{ m/s}$$

The friend’s pitching speed is 28 m/s.

(b) We have $v_{0y} = \pm v_0 \sin \theta$, where we will use the plus sign for up 5° and the minus sign for down 5° . We can write

$$y_1 = y_0 \pm v_0 \sin \theta (t_1 - t_0) - \frac{g}{2} (t_1 - t_0)^2 \Rightarrow 0 \text{ m} = 4 \text{ m} \pm v_0 \sin \theta t_1 - \frac{g}{2} t_1^2$$

Let us first find t_1 from $x_1 = x_0 + v_{0x}(t_1 - t_0)$:

$$25 \text{ m} = 0 \text{ m} + v_0 \cos \theta t_1 \Rightarrow t_1 = \frac{25 \text{ m}}{v_0 \cos \theta}$$

Now substituting t_1 into the y -equation above yields

$$0 \text{ m} = 4 \text{ m} \pm v_0 \sin \theta \left(\frac{25 \text{ m}}{v_0 \cos \theta} \right) - \frac{g}{2} \left(\frac{25 \text{ m}}{v_0 \cos \theta} \right)^2$$

$$\Rightarrow v_0^2 = \frac{g(25 \text{ m})^2}{2 \cos^2 \theta} \left\{ \frac{1}{4 \text{ m} \pm (25 \text{ m}) \tan \theta} \right\} = 22.3 \text{ m/s and } 44.2 \text{ m/s}$$

The range of speeds is 22 m/s to 44 m/s, which is the same as 50 mph to 92 mph.

Assess: These are reasonable speeds for baseball pitchers.

Problem 4.31

Peregrine falcons are known for their maneuvering ability. In a tight circular turn, a falcon can withstand a centripetal acceleration 1.5 times free-fall acceleration. What is the radius of turn if the falcon is flying at 25 m/s?

4.31. Visualize: The magnitude of centripetal acceleration is given by $a = \frac{v^2}{r}$.

Solve: The centripetal acceleration is given as 1.5 times the acceleration of gravity, so

$$a = (1.5)(9.80 \text{ m/s}^2) = 14.7 \text{ m/s}^2$$

The radius of the turn is given by

$$r = \frac{v^2}{a} = \frac{(25 \text{ m/s})^2}{14.7 \text{ m/s}^2} = 43 \text{ m}$$

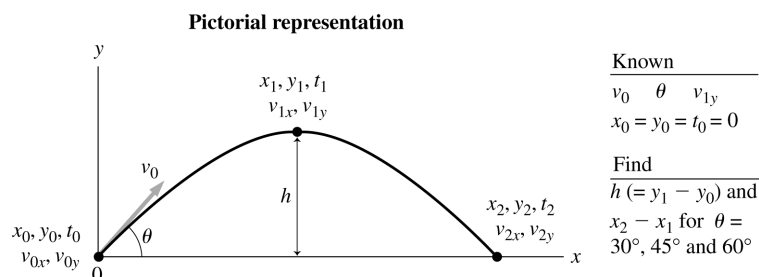
Assess: This seems reasonable.

Problem 4.47

(a) A projectile is launched with a speed v_0 and angle θ . Derive an expression for the projectile's maximum height. **(b)** A baseball is thrown with a speed of 33.6 m/s. Calculate its height and distance traveled if it is hit at angles of 30.0° , 45.0° , and 60.0° .

4.47. Model: Assume particle motion in a plane and constant-acceleration kinematics for the projectile.

Visualize:



Solve: (a) We know that $v_{0y} = v_0 \sin \theta$, $a_y = -g$, and $v_{1y} = 0$ m/s. Using $v_{1y}^2 = v_{0y}^2 + 2a_y(y_1 - y_0)$,

$$0 \text{ m}^2/\text{s}^2 = v_0^2 \sin^2 \theta + 2(-g)h \Rightarrow h = \frac{v_0^2 \sin^2 \theta}{2g}$$

(b) Using the equation for range and the above expression for $\theta = 30.0^\circ$:

$$h = \frac{(33.6 \text{ m/s})^2 \sin^2 30.0^\circ}{2(9.8 \text{ m/s}^2)} = 14.4 \text{ m}$$

$$(x_2 - x_0) = \frac{v_0^2 \sin 2\theta}{g} = \frac{(33.6 \text{ m/s})^2 \sin(2 \times 30.0^\circ)}{(9.8 \text{ m/s}^2)} = 99.8 \text{ m}$$

For $\theta = 45.0^\circ$:

$$h = \frac{(33.6 \text{ m/s})^2 \sin^2 45.0^\circ}{2(9.8 \text{ m/s}^2)} = 28.8 \text{ m}$$

$$(x_2 - x_0) = \frac{(33.6 \text{ m/s})^2 \sin(2 \times 45.0^\circ)}{(9.8 \text{ m/s}^2)} = 115.2 \text{ m}$$

For $\theta = 60.0^\circ$:

$$h = \frac{(33.6 \text{ m/s})^2 \sin^2 60.0^\circ}{2(9.8 \text{ m/s}^2)} = 43.2 \text{ m}$$

$$(x_2 - x_0) = \frac{(33.6 \text{ m/s})^2 \sin(2 \times 60.0^\circ)}{2(9.8 \text{ m/s}^2)} = 99.8 \text{ m}$$

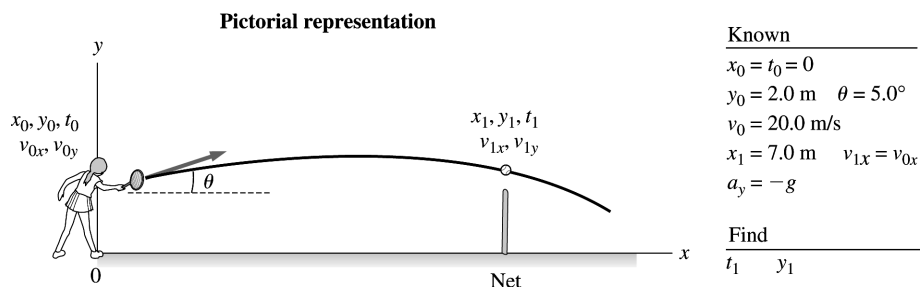
Assess: The projectile's range, being proportional to $\sin(2\theta)$, is maximum at a launch angle of 45° , but the maximum height reached is proportional to $\sin^2(\theta)$. These dependencies are seen in this problem.

Problem 4.51

A tennis player hits a ball 2.0 m above the ground. The ball leaves his racquet with a speed of 20.0 m/s at an angle of 5.0° above the horizontal. The horizontal distance to the net is 7.0 m, and the net is 1.0 m high. Does the ball clear the net? If so, by how much? If not, by how much does it miss?

4.51. Model: The particle model for the ball and the constant-acceleration equations of motion in a plane are assumed.

Visualize:



Solve: The initial velocity is

$$v_{0x} = v_0 \cos 5.0^\circ = (20 \text{ m/s}) \cos 5.0^\circ = 19.92 \text{ m/s}$$

$$v_{0y} = v_0 \sin 5.0^\circ = (20 \text{ m/s}) \sin 5.0^\circ = 1.743 \text{ m/s}$$

The time it takes for the ball to reach the net is

$$x_1 = x_0 + v_{0x}(t_1 - t_0) \Rightarrow 7.0 \text{ m} = 0 \text{ m} + (19.92 \text{ m/s})(t_1 - 0 \text{ s}) \Rightarrow t = 0.351 \text{ s}$$

The vertical position at $\vec{v} = \vec{v}' + \vec{V}$ is

$$\begin{aligned} y_1 &= y_0 + v_{0y}(t_1 - t_0) + \frac{1}{2}a_y(t_1 - t_0)^2 \\ &= (2.0 \text{ m}) + (1.743 \text{ m/s})(0.351 \text{ s} - 0 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(0.351 \text{ s} - 0 \text{ s})^2 = 2.01 \text{ m} \end{aligned}$$

Thus the ball clears the net by $1.01 \text{ m} \approx 1.0 \text{ m}$.

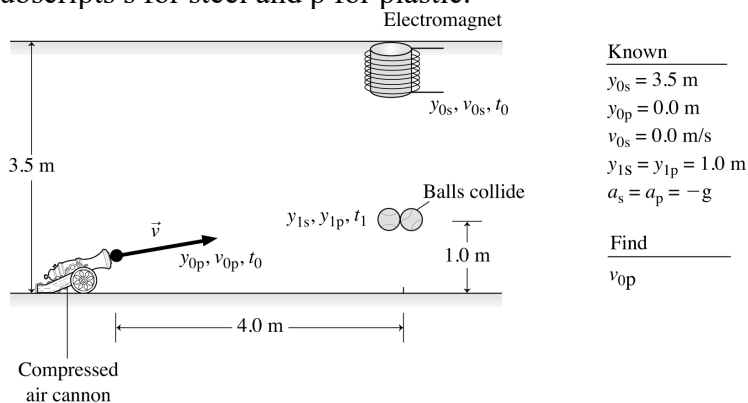
Assess: The vertical free fall of the ball, with zero initial velocity, in 0.351 s is 0.6 m. The ball will clear by approximately 0.4 m it is thrown horizontally. The initial launch angle of 5° provides some initial vertical velocity and the ball clears by a larger distance. The above result is reasonable.

Problem 4.53

A 35-g steel ball is held by ceiling-mounted electromagnet 3.5 m above the floor. A compressed-air cannon sits on the floor, 4.0 m to one side of the point directly under the ball. When a button is pressed, the ball drops and, simultaneously, the cannon fires a 25-g plastic ball. The two balls collide 1.0 m above the floor. What was the launch speed of the plastic ball?

4.53. Model: Both balls are particles in projectile motion with no air resistance. The steel ball is in one-dimensional free fall and the plastic ball is in two-dimensional projectile motion.

Visualize: Use subscripts s for steel and p for plastic.



Solve:

$$y_s = y_{0s} + v_{0s} t + \frac{1}{2}a_s t^2$$

$$y_p = y_{0p} + (v_{y0})_p t + \frac{1}{2}a_p t^2$$

Insert the known quantities and we have a system of two equations with two unknowns.

$$y_s = (3.5 \text{ m}) - \frac{1}{2}gt^2$$

$$y_p = (v_{y0})_p t - \frac{1}{2}gt^2$$

The balls meet at $y_{1s} = y_{1p} = 1.0 \text{ m}$. Find the time t_1 from the equation for the steel ball.

$$1.0 \text{ m} = (3.5 \text{ m}) - \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{5.0 \text{ m}}{9.8 \text{ m/s}^2}} = 0.7143 \text{ s}$$

Use this time in the vertical motion equation for the plastic ball.

$$1.0 \text{ m} = v_{0p}(0.7143 \text{ s}) - \frac{1}{2}g(0.7143 \text{ s})^2 \Rightarrow (v_{y0})_p = 4.9 \text{ m/s}$$

The plastic ball also moves horizontally at a constant rate:

$$x_p = x_{0p} + (v_{x0})_p t_1 \Rightarrow (v_{x0})_p = x_p / t_1 = (4.0 \text{ m}) / (0.7143 \text{ s}) = 5.6 \text{ m/s}$$

We use the Pythagorean theorem to find the initial speed of the ball.

$$v_{0p} = \sqrt{(v_{x0})_p^2 + (v_{y0})_p^2} = \sqrt{(5.6 \text{ m/s})^2 + (4.9 \text{ m/s})^2} = 7.4 \text{ m/s}$$

Assess: 7.4 m/s seems like a reasonable speed for an air cannon to launch a plastic ball. The masses of the balls did not matter.

Problem 4.A

An electron's position is given by $\vec{r} = (3.00t)\hat{i} - (4.00t^2)\hat{j} + 2.00\hat{k}$, with t in seconds and \vec{r} in meters. **(a)** In unit-vector notation, what is the electron's velocity $\vec{v}(t)$? At $t = 2.00 \text{ s}$, what is \vec{v} **(b)** in unit-vector notation and as **(c)** a magnitude and **(d)** an angle relative to the +x-axis.

note: $\vec{r}(t)$ should be written as:

$$\vec{r}(t) = [(3.00 \text{ m/s})t]\hat{i} - [(4.00 \text{ m/s}^2)t^2]\hat{j} + (2.00 \text{ m})\hat{k}$$

(a) $\vec{v} = d\vec{r}/dt = \boxed{(3.00 \text{ m/s})\hat{i} - [(8.00 \text{ m/s}^2)t]\hat{j}}$

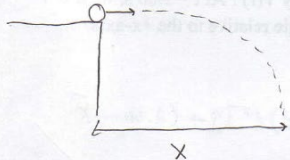
(b) $\vec{v}(t=2.00 \text{ s}) = (3.00 \text{ m/s})\hat{i} - (8.00 \text{ m/s}^2)(2.00 \text{ s})\hat{j}$
 $= \boxed{(3.00 \text{ m/s})\hat{i} - (16.0 \text{ m/s})\hat{j}}$

(c) $|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(3.00 \text{ m/s})^2 + (-16.0 \text{ m/s})^2} = \boxed{16.3 \text{ m/s}}$

(d) $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-16.0 \text{ m/s}}{3.00 \text{ m/s}}\right) \rightarrow \boxed{\theta = -79.4^\circ}$

Problem 4.B

A small ball rolls horizontally off the edge of a tabletop that is 1.20 m high. It strikes the floor at a point 1.52 m horizontally from the table edge. **(a)** How long is the ball in the air? **(b)** What is its speed at the instant it leaves the table?



$y_0 = 0 \text{ m}$
 $y = -1.20 \text{ m}$
 $v_{0y} = 0 \text{ m/s}$
 $v_y = ?$
 $a_y = -9.80 \text{ m/s}^2$
 $t = ?$

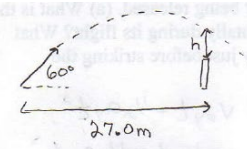
$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$
 $y = \frac{1}{2}a_y t^2$
 $t = \sqrt{\frac{2y}{a_y}}$
 $t = \sqrt{\frac{2(-1.20 \text{ m})}{-9.80 \text{ m/s}^2}}$
 $t = 0.495 \text{ s}$

b) $x_0 = 0 \text{ m}$
 $x = 1.52 \text{ m}$
 $v_{0x} = ?$
 $t = 0.495 \text{ s}$

$x - x_0 = v_{0x}t$
 $x = v_{0x}t$
 $v_{0x} = x/t = \frac{1.52 \text{ m}}{0.495 \text{ s}}$
 $v_{0x} = 3.07 \text{ m/s}$

Problem 4.C

A rocket is fired at a speed of 75.0 m/s from ground level, at an angle of 60.0° above the horizontal. The rocket is fired toward an 11.0 m high wall, which is located 27.0 m away. By how much does the rocket clear the top of the wall?



\Rightarrow we want y when $x = 27.0\text{ m}$
 \downarrow find t when $x = 27.0\text{ m}$; use t to find y

$X_0 = 0\text{ m}$
 $X = 27.0\text{ m}$
 $V_{0x} = (75.0\text{ m/s}) \cos 60^\circ = 37.5\text{ m/s}$
 $t = ?$

$X - X_0 = V_{0x}t \rightarrow X = V_{0x}t \rightarrow t = X/V_{0x}$
 $t = \frac{27.0\text{ m}}{37.5\text{ m/s}} \rightarrow \boxed{t = 0.72\text{ s}}$

$y_0 = 0\text{ m}$
 $y = ?$
 $V_{0y} = (75.0\text{ m/s}) \sin 60^\circ = 65.0\text{ m/s}$
 $a_y = -9.80\text{ m/s}^2$
 $t = 0.72\text{ s}$

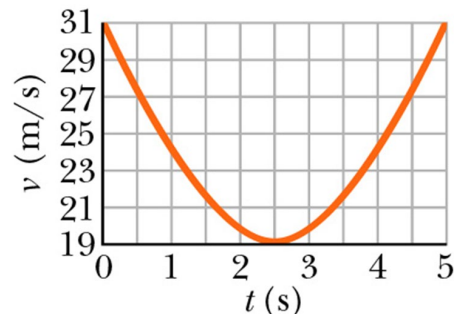
$y - y_0 = V_{0y}t + \frac{1}{2}a_yt^2 \rightarrow y = V_{0y}t + \frac{1}{2}a_yt^2$
 $y = (65.0\text{ m/s})(0.72\text{ s}) - \frac{1}{2}(-9.80\text{ m/s}^2)(0.72\text{ s})^2$
 $y = 44.3\text{ m}$ $h = 44.3\text{ m} - 11.0\text{ m} = 33.3\text{ m}$

rocket clears wall by 33.3 m

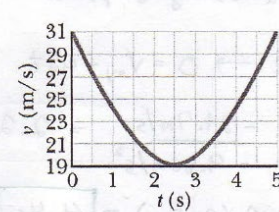
Problem 4.D

A golf ball is struck at ground level. The speed of the golf ball as a function of the time is shown in the figure below, where $t = 0$ at the instant the ball is struck.

(a) How far does the golf ball travel horizontally before returning to ground level? (b) What is the maximum height above ground level attained by the ball?



ground level attained by the ball:



\Rightarrow the velocity is a minimum at the highest point where $V_y = 0\text{ m/s}$; at the highest point, $V = V_{0x}$
 \Rightarrow from the graph, $V_{0x} = 19\text{ m/s}$ and the total time in the air is 5.0 s

$X - X_0 = V_{0x}t \rightarrow X = V_{0x}t$
 $X = (19\text{ m/s})(5.0\text{ s}) \rightarrow \boxed{X = 95\text{ m}}$

(a) $X_0 = 0\text{ m}$ $V_{0x} = 19\text{ m/s}$
 $X = ?$ $t = 5.0\text{ s}$

(b) from $V_0 = 31\text{ m/s} = \sqrt{V_{0x}^2 + V_{0y}^2}$, we can find $V_{0y} = 24.5\text{ m/s}$

$y_0 = 0\text{ m}$ $t = 2.5\text{ s}$ to highest point
 $y = ?$

$V_{0y} = 24.5\text{ m/s}$
 $a_y = -9.80\text{ m/s}^2$

$y - y_0 = V_{0y}t + \frac{1}{2}a_yt^2$
 $y = V_{0y}t + \frac{1}{2}a_yt^2$
 $y = (24.5\text{ m/s})(2.5\text{ s}) + \frac{1}{2}(-9.80\text{ m/s}^2)(2.5\text{ s})^2 = \boxed{31\text{ m}}$

Problem 4.E

(a) What is the magnitude of the centripetal acceleration of an object on Earth's equator owing to the rotation of Earth? (b) What would the period of rotation of Earth have to be for objects on the equator to have a centripetal acceleration with a magnitude of 9.8 m/s^2 ?

$$r_E = 6.37 \times 10^6 \text{ m}$$

$$T = 1 \text{ day} \left(\frac{24 \text{ hrs}}{1 \text{ day}} \right) \left(\frac{3600 \text{ s}}{1 \text{ hr}} \right) = 86,400 \text{ s}$$

$$a) \quad v = \frac{2\pi r}{T} = \frac{2\pi(6.37 \times 10^6 \text{ m})}{86,400 \text{ s}} \longrightarrow \boxed{v = 463 \text{ m/s}}$$

$$b) \quad a = \frac{v^2}{r} \quad v = \frac{2\pi r}{T} \longrightarrow a = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$T = \frac{4\pi^2 r}{a} \longrightarrow T = \sqrt{\frac{4\pi^2 r}{a}} = \sqrt{\frac{4\pi^2(6.37 \times 10^6 \text{ m})}{9.80 \text{ m/s}^2}}$$

$$\boxed{T = 5.1 \times 10^3 \text{ s} \approx 84 \text{ min}}$$